

On instability of hadronic string with heavy quarks at its ends

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Abstract

The quark mass dependence of the energy spectrum in the Nambu–Goto string with point-like masses (quarks) at its ends is analyzed. To this end, linearized equations of motion and boundary conditions in this model are considered. It is shown that for sufficiently large quark masses, the first excited state in string spectrum may be arbitrary close to the ground state. Obviously this points to infrared instability in the system under consideration. Possible modifications of string model which could remove this drawback are discussed.

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1 Introduction

Investigation of the quark dynamics inside hadrons remains as before the most important task of theoretical and experimental studies in hadronic physics. In view of asymptotic freedom in QCD, this problem at small distances can be treated in the framework of perturbative calculations. However at distances like hadron size R_h and larger nonperturbative effects become important. Therefore in this region lattice simulations and string models are used for revealing the interquark interaction (for a review and further references, see, e.g. [1], [2]).

Usually one assumes that at distances $\sim R_h$ the most important are such configurations of gluonic field when this field is concentrated along the line connecting quarks (the flux tube between quarks). Such infinitely thin flux tube is simulated by relativistic string. In the most simple case, the dynamics of this system is determined by the Nambu–Goto string action [3]. However other string models were also proposed to describe quark interaction, for example Polyakov–Kleinert rigid string [4], [5]. Search for new string models is stimulated by comparing the calculations in QCD with those in the framework of known string models [6].

A large body of literature is devoted to the calculation of the interquark potential in string models (see, for example, [7]–[11] and references therein). The central point in these calculations is a determination of the string energy as a function of the distance between quarks. When deconfinement temperature (or critical temperature) is investigated then the dependence of the effective string tension on temperature should be calculated. Practically in all these investigations only the static interquark potential has been considered¹. It means that the ends of the string connecting quarks are fixed. On the one hand, this assumption essentially simplifies the problem because the string frequencies in this case are integer. On the other hand, results obtained in this way are applied, strictly speaking, only to the infinitely heavy quarks which cannot move. However in this case the notion of the interquark potential practically loses its usefulness.

As a matter of fact static string potential is treated, after its derivation, as a usual potential describing the interaction between quarks with finite masses. Here it is implicitly assumed that the interquark potential is a smooth function of quark mass m when $m \rightarrow \infty$. Probably it is right. Calculation of the interquark potential generated by string with massive ends carried out in our recent paper [13] testifies to this conclusion. Nevertheless we would like to draw attention to one peculiarity in energy spectrum of the relativistic string with heavy quarks at its ends which is, to our view, very important. The point is

¹First attempt to calculate the quark mass dependence of the string potential was made in Ref. [12]. The authors are grateful to Prof. H. Kleinert for providing them with this paper.

that the position of the first excited level in this system depends on the quark mass m and when $m \rightarrow \infty$ this level may be arbitrary close to the ground state. Obviously this testifies to the infrared instability of the string connecting quarks with sufficiently large masses. It is this problem that will be discussed in our short communication. The layout of the paper is the following. In Section 2 the basic facts from the classical and quantum theories of the Nambu–Goto string with massive ends are given. Then linearization of equations of motion and boundary conditions is considered. It should be noted here that it is these linearized equations that are used as a starting point in all the calculations of the interquark potential in string models both in perturbation theory and by making use of the functional integration (variational estimations). Therefore the spectrum of eigenfrequencies in this system is of great importance. In Section 3 the quark mass dependence of the string spectrum is investigated. It is shown that the energy of the first excited level, E_1 , may be arbitrary close to the ground state with energy E_0 when quark mass tends to infinity. This is a direct indication about the infrared instability in the system under consideration. In Conclusion (Section 4) the physical implications of this result and possible modifications of the string model for removing this drawback are shortly discussed.

2 Equations of motion and boundary conditions in linear approximation

The Nambu–Goto string with point–like masses attached to its ends is described by action [3]

$$S = -M_0^2 \iint_{\Sigma} d\Sigma - \sum_{a=1}^2 m_a \int_{C_a} ds_a, \quad (2.1)$$

where $d\Sigma$ is infinitesimal area of the string world surface, C_a , $a = 1, 2$, are the world trajectories of the string massive ends, m_a , $a = 1, 2$, are the quark masses ($\hbar = c = 1$). M_0^2 is the string tension. In hadronic physics one usually sets $M_0 \sim 1$ GeV.

The Gauss parametrization of the string world surface embedded into D –dimensional space–time with signature $(+, -, \dots, -)$

$$\begin{aligned} x^\mu(t, r) &= (t, r; x^1(t, r), \dots, x^{D-2}(t, r)) = \\ &= (t, r; \mathbf{u}(t, r)), \end{aligned} \quad (2.2)$$

permits us to write the induced metric on this surface, $g_{ij} = \partial_i x^\mu \partial_j x_\mu$, in the following way

$$g_{ij} = \delta_{ij} - \mathbf{u}_i \mathbf{u}_j, \quad i, j = 0, 1. \quad (2.3)$$

Here $\mathbf{u}_0 = \partial_0 \mathbf{u} = \partial \mathbf{u} / \partial t = \dot{\mathbf{u}}$, $\mathbf{u}_1 = \partial_1 \mathbf{u} = \partial \mathbf{u} / \partial r = \mathbf{u}'$ and $\mathbf{u} \mathbf{u} = \sum_{j=1}^{D-2} u^j u^j$. From Eq. (2.3) it follows that in the quadratic approximation the determinant g of the induced metric is given by

$$g = \det(g_{ij}) \simeq 1 - \mathbf{u}_i^2. \quad (2.4)$$

In this approximation, the infinitesimal area $d\Sigma$ can be written as $d\Sigma = \sqrt{g} dt dr \simeq [1 - \mathbf{u}_i^2/2] dt dr$, while the line elements ds_a , $a = 1, 2$, take the form $ds_a = [1 - \dot{\mathbf{u}}^2(t, r_a)/2] dt$. Finally the action (2.1) becomes

$$S \simeq \frac{M_0^2}{2} \int_{t_1}^{t_2} dt \int_0^R dr [\dot{\mathbf{u}}^2(t, r) - \mathbf{u}'^2(t, r)] + \sum_{a=1}^2 \frac{m_a}{2} \int_{t_1}^{t_2} dt \dot{\mathbf{u}}^2(t, r_a), \quad (2.5)$$

$$r_1 = 0, \quad r_2 = R.$$

Equations of motion and boundary conditions for the system described by (2.5) can be immediately deduced

$$\square \mathbf{u}(t, r) = 0, \quad (2.6)$$

$$m \ddot{\mathbf{u}} = M_0^2 \mathbf{u}', \quad r = 0, \quad (2.7)$$

$$m \ddot{\mathbf{u}} = -M_0^2 \mathbf{u}', \quad r = R, \quad (2.8)$$

where $\square = \partial^2/\partial t^2 - \partial^2/\partial r^2$ and we put for simplicity $m_1 = m_2 = m$. The general solution to Eq. (2.6)–(2.8) is given by

$$u^j(t, r) = i \frac{\sqrt{2}}{M_0} \sum_{n \neq 0} \exp[-iM_0\omega_n t] \frac{\alpha_n^j}{\omega_n} u_n(r), \quad j = 1, \dots, D-2. \quad (2.9)$$

Amplitudes α_n^j satisfy the usual rule of complex conjugation $\alpha_n^* = \alpha_{-n}$. The eigenfunctions $u_n(r)$ in (2.9) are defined by

$$u_n(r) = N_n [\cos(\omega_n M_0 r) - \mu \omega_n \sin(\omega_n M_0 r)]. \quad (2.10)$$

N_n 's are normalization constants and μ is the dimensionless parameter

$$\mu = \frac{m}{M_0}. \quad (2.11)$$

The eigenfrequencies ω_n are the roots of the transcendental equation²

$$\tan(\omega_n M_0 R) = \frac{2\mu\omega_n}{\mu^2\omega_n^2 - 1}. \quad (2.12)$$

On the ω -axis these roots are placed symmetrically around zero. Hence they can be numbered in the following way $\omega_{-n} = -\omega_n$, $n = 1, 2, \dots$. Therefore it will be sufficient to consider only the positive roots. The Hamiltonian operator of the system under consideration in terms of creation and annihilation operator, a^+ and a respectively, reads

$$H = M_0 \sum_{n=1}^{\infty} \sum_{j=1}^{D-2} \omega_n a_n^{j+} a_n^j + \frac{D-2}{2} M_0 \sum_{n=1}^{\infty} \omega_n, \quad (2.13)$$

where

$$[a_n^i, a_m^{j+}] = \delta^{ij} \delta_{nm}, \quad n, m = 1, 2, \dots$$

The last term in Eq. (2.13) is the Casimir energy [16], [17].

²This equation is well known in mathematical physics. Besides the problem about linear vibrations of the string with point-like masses at its ends, this equation determines eigenfrequencies of the torsional vibrations of shaft with massive discs at its ends [14] and eigenvalues in the boundary value problem about heat flow along rod under special conditions at its ends [15].

3 Quark mass dependence of the string spectrum

In this Section we investigate the behaviour of the energy levels in string spectrum at large quark masses. To this end, let us separate the integer part in each root of Eq. (2.12)

$$M_0 R \omega_n = (n-1)\pi + \varepsilon_n, \quad 0 < \varepsilon_n < \pi, \quad n = 1, 2, \dots, \quad (3.1)$$

where the correction ε_n to the frequencies ω_n depends on quark masses. By substituting Eq. (3.1) into Eq. (2.12), one finds, for arbitrary n and in the limit $\mu \rightarrow \infty$, that ε_n satisfies the following equation

$$\tan \varepsilon_n \simeq \frac{2M_0 R}{(n-1)\pi + \varepsilon_n} \mu^{-1}. \quad (3.2)$$

It means that $\varepsilon_n \rightarrow 0$ when $\mu \rightarrow \infty$. Solution to Eq. (3.2) for $n = 1$ is

$$\varepsilon_1 \simeq \sqrt{\frac{2M_0 R}{\mu}}, \quad (3.3)$$

For $n > 1$ we have

$$\varepsilon_n \simeq \frac{2M_0 R}{\mu(n-1)\pi}. \quad (3.4)$$

Energy levels of the system are calculated by making use of Eq. (2.13). The energy of the first excited state, E_1 , is

$$E_1 = M_0 \omega_1 = \frac{\varepsilon_1}{R} \simeq M_0 \sqrt{\frac{2}{M_0 R \mu}}. \quad (3.5)$$

For fixed string tension M_0 and string length R , the energy E_1 decreases for raising values of the dimensionless parameter μ . From physical point of view, this behaviour of E_1 implies the instability of the string with heavy quarks at its ends.

Figure 1 shows the dependence of the dimensionless energy of the first excited state $E_1/M_0 = \omega_1$ on the dimensionless quark mass $\mu = m/M_0$ (bold-face curve). This curve is obtained for the string length of order of the hadron size³ $M_0 R = 1$. When $\mu \sim 10^2$

³When calculating the interquark potential generated by string we have to consider arbitrary length of the string R . From frequency equation (2.12) one can easily deduce the following scaling property of its roots. Let the functions $f_n(\mu)$, $n = 1, 2, \dots$ determine the roots of this equation for given value of the mass parameter μ and for $M_0 R = 1$, i.e.

$$\omega_n(\mu, M_0 R = 1) = f_n(\mu).$$

Then for arbitrary string length $M_0 R \neq 1$ the following scaling rule holds

$$\omega_n(\mu, M_0 R \neq 1) = \frac{1}{M_0 R} f_n\left(\frac{\mu}{M_0 R}\right).$$

(region of the top quark mass) we have $E_1/M_0 \sim 0.1$. There are no any restrictions on the number of excitations with energy E_1 . Therefore, we really have infinite sequence of equidistant energy levels $E_n = n E_1$, $n = 1, 2, \dots$, which is superimposed on the basic string spectrum with scale M_0 (see Eq. (2.13)). As a consequence, for sufficiently large quark masses the ground state of the string becomes fuzzy, i.e. weak, compared with M_0 , external perturbations will result in excitation of the string (see Fig. 1). Obviously this implies the infrared instability in this system.

It should be noted that all these considerations are applicable to the rigid string too. In fact, in Ref. [13] the same frequency equation as (2.11) was derived in the model of the rigid string with massive ends.

Where may this infrared instability be manifested? At first, when going beyond the linear approximation in string model. To this end we have to take into account the "interaction" in the string action (2.4) which is described by terms $\sim \mathbf{u}^n$ with $n > 2$. Besides, this infrared instability may be revealed in calculations at finite temperature. For example, effective string potential generated by linearized string action (2.4) is obtained by functional integration over "field" $\mathbf{u}(t, r)$. As a result, one arrives at the following well known expression [18]

$$\sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} \ln(\bar{\omega}_m^2 + k_n^2) = \frac{2}{T} \sum_{n=1}^{\infty} \left[\frac{k_n}{2} + T \ln(1 - e^{-k_n/T}) \right], \quad (3.6)$$

where $\bar{\omega}_m = 2\pi T m$ are the Matsubara frequencies and $k_n/M_0 = \omega_n$ are the roots of Eq. (2.12). When $m \rightarrow \infty$, $k_1 \rightarrow 0$ and we obtain infinity in (3.7). On the other hand, calculations in string models with fixed ends are well defined at finite temperature [19].

4 Conclusion

The instability of the relativistic string, connecting heavy quarks is quiet clear because fixed string tension M_0^2 proves to be insufficient to keep in bound state very heavy quarks. In the framework of the string approach to hadron physics this instability apparently implies that string-like collective excitations of gluon field do not dominate in heavy quark dynamics. Though other points of view can be proposed. For example, one way suppose that string tension for heavy quarks should be greater than M_0^2 . However the following problem arises here: in what way this assumption can be put in agreement with basic concepts of QCD.

In the same way one may doubt in applicability of the linearized string action (2.5) which has been used to derive frequency equation (2.12). But in this case it would be difficult to understand why this action is good for description of the string with light quarks at its ends or for fixed string ends but is not applicable to heavy quarks. Proceeding from simple physical considerations one may expect that situation should be opposite because linearization of the string action is, in some sense, equivalent to the nonrelativistic approximation (see, for example, [20]).

Thus the problem of infrared instability in the dynamics of the relativistic string connecting sufficiently heavy quarks remains, in our opinion, open.

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Figure Caption

Fig. 1. Quark mass dependence of the energy levels in string spectrum. The bold-faced curve is the dimensionless energy of the first excited state, E_1/M_0 ; $\mu = m/M_0$ is dimensionless quark mass. Other curves present the next 5 energy levels, $E_n/M_0 = n E_1/M_0$, $n = 2, 3, \dots, 6$.

